

Différentielle - corrigé des exercices

Exercice 1.

1. $f(x_1, x_2) = 5x_1 - 4x_2 + 7$

Corrigé : $f_1 = 5, f_2 = -4.$

2. $f(x_1, x_2) = 3x_1^2 + 2x_1x_2 + x_2^3$

Corrigé : $f_1 = 6x_1 + 2x_2, f_2 = 2x_1 + 3x_2^2.$

3. $f(x_1, x_2) = 5x_1 - 4x_2 + 7$

Corrigé : $f_1 = 5, f_2 = -4.$

4. $f(x_1, x_2) = x_1^3 + 4x_1x_2^2 - 2x_2^4$

Corrigé : $f_1 = 3x_1^2 + 4x_2^2, f_2 = 8x_1x_2 - 8x_2^3.$

Exercice 2.

1. $f(x_1, x_2) = \frac{x_1 + x_2}{x_1 - x_2}$

Corrigé : $f_1 = \frac{(x_1 - x_2) - (x_1 + x_2)}{(x_1 - x_2)^2} = \frac{-2x_2}{(x_1 - x_2)^2}, f_2 = \frac{(x_1 - x_2) + (x_1 + x_2)}{(x_1 - x_2)^2} = \frac{2x_1}{(x_1 - x_2)^2}.$

2. $f(x_1, x_2) = e^{x_1+2x_2} \cdot \ln(x_1)$

Corrigé : $f_1 = e^{x_1+2x_2} \cdot \ln(x_1) + \frac{e^{x_1+2x_2}}{x_1}, f_2 = 2e^{x_1+2x_2} \cdot \ln(x_1).$

3. $f(x_1, x_2) = \sin(x_1^2 + x_2) \cdot \cos(x_2)$

Corrigé : $f_1 = 2x_1 \cos(x_1^2 + x_2) \cos(x_2), f_2 = \cos(x_1^2 + x_2) \cos(x_2) - \sin(x_1^2 + x_2) \sin(x_2).$

4. $f(x_1, x_2) = (x_1^2 + x_2^2)^{\frac{1}{3}}$

Corrigé : $f_1 = \frac{2x_1}{3(x_1^2 + x_2^2)^{\frac{2}{3}}}, f_2 = \frac{2x_2}{3(x_1^2 + x_2^2)^{\frac{2}{3}}}.$

Exercice 3.

1. $f(x_1, x_2) = x_1^2 + x_1x_2 - 5x_2^2$

Corrigé : $df = (2x_1 + x_2)dx_1 + (x_1 - 10x_2)dx_2.$

2. $f(x_1, x_2) = x_1^2 - x_2 \ln(x_1)$

Corrigé : $df = \left(2x_1 - \frac{x_2}{x_1}\right) dx_1 - \ln(x_1)dx_2.$

3. $f(x_1, x_2) = \ln\left(\frac{x_1^2 + x_2^2}{x_1x_2}\right)$

Corrigé : $df = \left(\frac{2x_1}{x_1^2 + x_2^2} - \frac{1}{x_1}\right) dx_1 + \left(\frac{2x_2}{x_1^2 + x_2^2} - \frac{1}{x_2}\right) dx_2.$