

Exercise 5

$$1) \begin{vmatrix} 1 & \sin x^2 & \cos x^2 \\ 1 & \sin y^2 & \cos y^2 \\ 1 & \sin z^2 & \cos z^2 \end{vmatrix} = \begin{vmatrix} 1 & \sin x^2 + \cos x^2 & \cos x^2 \\ 1 & \sin y^2 + \cos y^2 & \cos y^2 \\ 1 & \sin z^2 + \cos z^2 & \cos z^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & \cos x^2 \\ 1 & 1 & \cos y^2 \\ 1 & 1 & \cos z^2 \end{vmatrix} = 0$$

2)

$$\begin{aligned} & \begin{vmatrix} 1 & \sin x & \cos x \\ 1 & \sin y & \cos y \\ 1 & \sin z & \cos z \end{vmatrix} = \begin{vmatrix} 1 & \sin x & \cos x \\ 0 & \sin y - \sin x & \cos y - \cos x \\ 0 & \sin z - \sin x & \cos z - \cos x \end{vmatrix} = \begin{vmatrix} \sin y - \sin x & \cos y - \cos x \\ \sin z - \sin x & \cos z - \cos x \end{vmatrix} \\ &= (\sin y - \sin x)(\cos z - \cos x) - (\sin z - \sin x)(\cos y - \cos x) \\ &= \sin y \cos z - \sin y \cos x - \sin x \cos z + \sin x \cos x - \sin z \cos y + \sin z \cos x + \sin x \cos y \\ &\quad - \sin x \cos x \\ &= \sin y \cos z - \sin z \cos y + \sin x \cos y - \sin y \cos x + \sin z \cos x - \sin x \cos z \\ &= \sin y - z + \sin x - y + \sin z - x \\ &= 2 \sin \frac{x-z}{2} \cos \frac{2y-z-x}{2} + \sin 2 \frac{z-x}{2} \\ &= 2 \sin \frac{x-z}{2} \cos \frac{2y-z-x}{2} - 2 \sin \frac{z-x}{2} \cos \frac{z-x}{2} \\ &= 2 \sin \frac{x-z}{2} \left( \cos \frac{2y-z-x}{2} - \cos \frac{z-x}{2} \right) \\ &= 2 \sin \frac{x-z}{2} \left( \sin \frac{y-x}{2} \sin \frac{y-z}{2} \right) = -2 \sin \frac{x-z}{2} \sin \frac{y-x}{2} \sin \frac{z-y}{2} \end{aligned}$$