Exercice n°4 :

- $$\begin{vmatrix} 1 & 1 & 1 \\ a+b & c+a & b+c \\ ab & ca & bc \end{vmatrix} \underset{\substack{C_2 \leftarrow C_2 - C_1 \\ C_3 \leftarrow C_3 - C_1}}{=} \begin{vmatrix} 1 & 0 & 0 \\ a+b & c-b & c-a \\ ab & ca-ab & bc-ab \end{vmatrix}$$

$$= \begin{vmatrix} c-b & c-a \\ a(c-b) & b(c-a) \end{vmatrix} = (c-b)(c-a) \begin{vmatrix} 1 & 1 \\ a & b \end{vmatrix}$$

$$= (c-b)(c-a)(b-a) \qquad \underline{\text{cqfd}}$$

- $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \underset{C_1 \leftarrow C_1 - C_3}{=} \begin{vmatrix} -a-b-c & 2a & 2a \\ 0 & b-c-a & 2b \\ c+a+b & 2c & c-a-b \end{vmatrix}$$

$$\underset{C_2 \leftarrow C_2 - C_3}{=} \begin{vmatrix} -a-b-c & 0 & 2a \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$$

$$\underset{L_2 \leftarrow L_2 + L_3}{=} \begin{vmatrix} -a-b-c & 0 & 2a \\ c+a+b & 0 & c-a+b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$$

$$+ \qquad - \qquad +$$

$$= -(c+a+b) \begin{vmatrix} -a-b-c & 2a \\ c+a+b & c-a+b \end{vmatrix}$$

$$= -(c+a+b)(c+a+b) \begin{vmatrix} -1 & 2a \\ 1 & c-a+b \end{vmatrix}$$

$$= -(c+a+b)^2 \left[-1 \times (c-a+b) - 2a \right]$$

$$= -(c+a+b)^2 \left[-c+a-b-2a \right]$$

$$= -(c+a+b)^2 (-a-b-c)$$

$$= (a+b+c)^3 \qquad \underline{\text{cqfd}}$$

Exercice n°5 :

1) $$\begin{vmatrix} 1 & \sin^2 x & \cos^2 x \\ 1 & \sin^2 y & \cos^2 y \\ 1 & \sin^2 x & \cos^2 z \end{vmatrix} \underset{C_2 \leftarrow C_2 + C_3}{=} \begin{vmatrix} 1 & 1 & \cos^2 x \\ 1 & 1 & \cos^2 y \\ 1 & 1 & \cos^2 z \end{vmatrix} = 0 \quad \text{car, } C_2 = C_3$$